Space Travel

There are estimated to be more than 10^{11} galaxies in the observable universe. They range in size from dwarfs with 10^7 stars to giants with 10^{12} stars. The Milky Way Galaxy is a typical spiral galaxy with over 10^{11} stars. The Sun is a fairly typical star located 7.62 ± 0.32 kpc (kiloparsecs) from the galactic center.

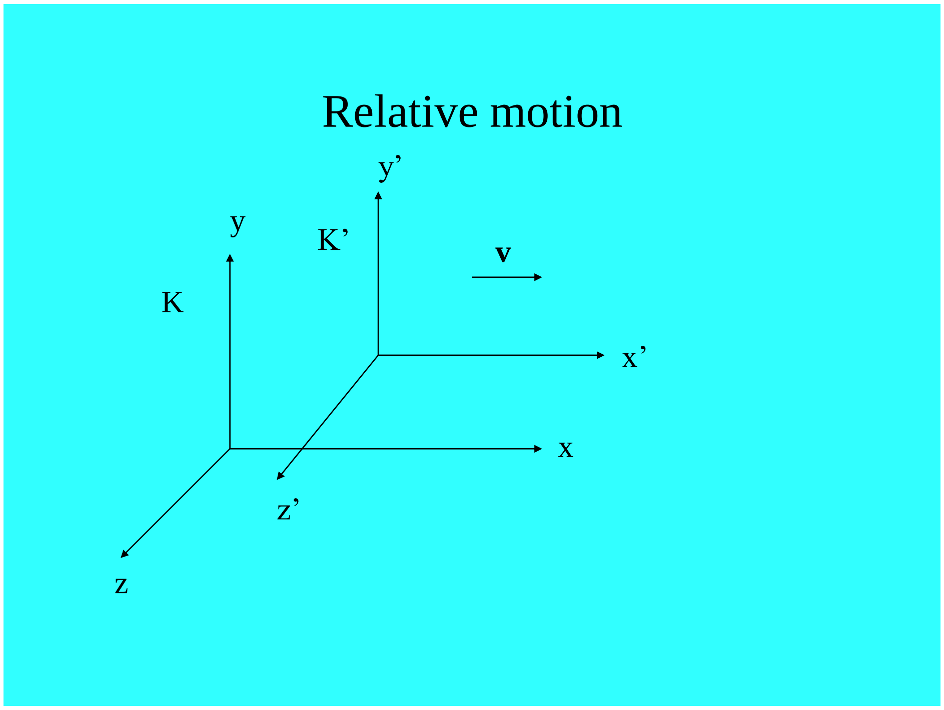
A parsec is 3.26 ly (light-years), and since a ly is defined as the distance light travels in vacuum in one Julian year (defined as exactly 365.25 days of 86,400 seconds each, totaling 31,557,600 seconds) at a speed of exactly 299,792,458 m/s (by definition) then a ly is exactly 9,460,730,472,580.8 km, or about 6 trillion miles. Thus a parsec is about 30.857×10^{15} m, about 19 trillion miles. This puts the Sun about 145,000 trillion miles (1.45×10^{17} miles) from the galactic center (or about 2.3×10^{20} m).

Most galaxies are 10^3 to 10^5 parsecs in diameter (the Milky Way Galaxy is 3×10^4 parsecs in diameter). Galaxies are usually separated by millions of parsecs. On the other hand the nearest star to the Sun is Proxima Centauri at 4.2 ly, a bit more than one parsec away.

Given these introductory facts, we will consider the requirements for travel to Proxima Centauri, to the galactic center and to a nearby galaxy. The basis for most of our considerations will be the Special Theory of Relativity. While the General Theory of Relativity is required for a deeper understanding, the Special Theory will suffice for making the main points and will be considerably more accessible to the lay reader. Even with respect to the Special Theory we will only give a few known results and no derivations.

The main point I will make is that space travel beyond our own Solar System creates extreme requirements that are extremely difficult to imagine ever satisfying, despite what so many science fiction books and movies may imply.

We begin by reviewing some of the basic results from Einstein's Special Theory of Relativity. The word *Special* refers to a restriction (lifted by the General Theory) to inertial frames of reference. These are frames of reference that move at constant velocity. What the theory is about is how observers measure and compare lengths and time intervals in reference frames in relative motion. Consider a frame K that is at rest with Cartesian coordinates *x*, *y* and *z* as depicted in the figure. Relative to frame K a frame K' with Cartesian coordinates *x'*, *y'* and *z'* moves along the positive x-axis with velocity v. Note that the *x'*-axis and the *x*-axis are parallel.



Einstein used the principle that the speed of light, c, is an absolute invariant (the same with respect to any inertial frame) and the equivalence of inertial frames of reference for expressing the laws of physics to derive the Lorentz transformations that determine how to translate results in K for lengths and time intervals into results in K'. These transformations are given by (notice that time appears in the combination *ct* because this has the same physical dimensions as *x*, *y*, and *z*, i.e. length, thereby imparting a nice symmetry to the expressions):

Lorentz Transformations

$$x' = \gamma\left(x - \frac{\mathrm{v}}{c}ct\right)$$
$$y' = y$$
$$z' = z$$
$$ct' = \gamma\left(ct - \frac{\mathrm{v}}{c}x\right)$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{\mathrm{v}}{c}$$

The importance of the ratio of the velocity of relative motion to the speed of light, β = v/c, is evident and the abbreviation γ is used in many places in relativistic expressions.

In taking the earlier point of view of Galilean Relativity, expressed by:

Galilean Invariance

$$x' = x - \mathrm{v}t$$
$$y' = y$$
$$z' = z$$
$$(t' = t)$$

time is absolute. This aspect was taken for granted before Einstein. It is clear from the expressions that the formulae for the Lorentz transformations reduce to the Galilean transformation expressions when $\beta << 1$. Only when a massive object moves at a speed very close to the speed of light, i.e. $\beta \sim 1$, do important differences arise. Because of the form for γ, how close to 1 must v/c be to be of interest is not intuitively obvious. What is found is that the successive values for v/c of 0.90…, 0.990…, 0.9990…, … are where the interest lies, and not for smaller values of β.

"Why is this?"

"Because we subtract the square of β from 1 in the denominator of γ."

Using elementary arithmetic, note that

if v/c = .999…90000… in which there are *n* 9's, followed by nothing but zeros **then** β^2 = .99…980…010…. in which there are *n-1* 9's initially, then the 8, and then *n-1* zeros, a 1 and then nothing but zeros.

For an error of only $1/10^n$ we can neglect the lone 1 with impunity. The result is summarized by:

$$\gamma = \frac{1}{\sqrt{2}} \times 10^{n/2}$$

For *n* = 1, $\gamma = \sqrt{5}$. Thus, even at 90% the speed of light the γ-factor is a bit over 2.236. At 99%, or *n* = 2, $\gamma = \frac{10}{\sqrt{2}}$ ~ 7 hardly much greater. It takes lot of 9's, or big *n*, to affect a really big difference.

For space travel a really big difference is needed to travel the great distances in relatively short times. Going to Proxima Centauri at 99% the speed of light could be done in somewhat more than 4 years, but the center of the galaxy would take about 25,000 years, at least from the viewpoint of the Earth. Special Relativity suggests a potential way out. Pick a frame K at rest with respect to the

Earth and Proxima Centauri (there is probably a non-zero relative motion between the Earth and Proxima Centauri that we ignore as small compared to the relative velocity, v, of the frames. Frame K' will be the frame attached to the spaceship in which we travel.

"How does the trip look in frame K?"

"Let the position of the Earth in K be x_1 and the position of Proxima Centauri be x_2. The trip starts at t_1 and ends at t_2. Since we travel at velocity v in K the distant traveled is related to the time duration by

$$(x_2 - x_1) = v(t_2 - t_1)\text{"}$$

"How does the trip look in frame K'?"

"We used the Lorentz transformations to obtain

$$(x_2' - x_1') = \gamma\big((x_2 - x_1) - \beta c(t_2 - t_1)\big)$$

$$c(t_2' - t_1') = \gamma\big(c(t_2 - t_1) - \beta(x_2 - x_1)\big)$$

The first equation can be evaluated using the equation from frame K and the right-hand side vanishes. That means $x_2' = x_1'$ which makes sense since K' is stationary relative to the spaceship. The spaceship is always at the same x' position in K' throughout the trip. The second K' equation tells us how long the trip took in K'. The result of again using the equation from frame K is

$$c(t_2' - t_1') = \gamma\big(c(t_2 - t_1) - \beta v(t_2 - t_1)\big) = \gamma\frac{1}{\gamma^2}c(t_2 - t_1) = \frac{1}{\gamma}c(t_2 - t_1)$$

For big γ the apparent duration of the trip, i.e. from the point of view of K', is shorter than determined in K."

[This result is the same as would be obtained by the "time dilation of a moving clock" if the clock is on the spaceship and is compared to clocks on the Earth. Such an effect is amply and precisely verified, for moving elementary particles with time dilated decay lifetimes, using the big accelerators at Fermi Lab, CERN, SLAC, etc..]

As an example, consider going to Proxima Centauri at 0.99c. In K it will take slightly more than 4.2 years. In K' it takes only $\frac{4.3}{7}$ years, about 7 months. This means a space traveler could go to Proxima Centauri to look for planets like the Earth and return in, say, 14-15 months from his/her perspective, while his/her relatives and associates back on Earth would have aged only about 9 years. Because Proxima Centauri is a red dwarf, it is very unlikely that the trip would be successful (no Earth-like planets). As a cautionary note, do not become confused by the so-called "twin paradox" in which it is alleged that there is the perspective that the space traveler stayed still while the Earth receded and returned instead, so that the Earthlings appear to be younger after the trip. This isn't a real *paradox* but is instead a *fallacy* based on the *lack of symmetry* between the space traveler and the Earthlings. When the space traveler reaches Proxima Centauri he/she must turn around to return to Earth and that involves deceleration/acceleration which makes the spaceship frame of reference non-inertial for awhile. The Earth frame is always inertial during the entire trip. When a proper account of acceleration is made using the equivalence principle or elementary General Relativity, both the space traveler and the Earthlings will agree that the space traveler is younger upon return. *While remarkable it means time travel to the future is possible, but not to the past.*

Now consider travel to the galactic center (for those who posit a black hole at the center of the Milky Way Galaxy, and who posit that with a black hole travel to greatly remote parts of the Universe in human time is possible, these requirements are especially pertinent). As stated above, a speed of .99c requires 25,000 years as observed from the Earth. Even the space traveler experiences a duration of 25,000/7 years (3,571 years). This is unacceptable for the traveler (we do not have any way to use "suspended animation" on humans for such lengths of time). If the traveler desires to make the trip during his natural lifetime, say, in 10 years, then $\gamma = 2{,}500$ is required. Using the γ-formula above for *n* 9's we find that *n*

= 7 is pretty close to what is needed yielding $\gamma = \sqrt{5} \times 10^3$. This means we need v = 0.9999999c, seven 9's, i.e. 99.99999% the speed of light. Such speeds are easily attainable for elementary particles in big accelerators such as at Fermi Lab or at CERN, but for a human being, attainment is entirely another story.

That other story, just alluded to, has to do with mass. As we have seen space and time are transformed into each other by the Lorentz transformations. The quantities *x*, *y*, *z* and *ct* form the components of what is called a four-vector in the Special Theory of Relativity. Other physical quantities make up the components of other four-vectors. Indeed, Einstein's postulate that the laws of physics are invariant under changes of inertial frames of reference leads to the notion that all physical quantities are components of four-vectors or tensors of higher order. It was Einstein's teacher, Hermann Minkowski, who made this idea especially clear. As it turns out, the three components of linear momentum and the energy of a particle make up the four components of a four-vector. This results in a surprising effect that says that the energy of a particle depends on its velocity of motion according to the mass-energy formula:

$$E = \gamma m_0 c^2$$

in which m_0 denotes the mass of the particle at rest, i.e. when v = 0, and is called the rest mass. If the velocity of motion is small compared to the speed of light then γ can be approximated to leading order by

$$\gamma \sim 1 + \frac{v^2}{2c^2}$$

This yields the approximation for non-relativistic velocities:

$$E \sim m_0 c^2 + \frac{1}{2} m_0 v^2$$

The second term on the right is the non-relativistic kinetic energy of Newton. But there is also a rest mass energy term given by the first term. In much of physics we deal with energy differences and this rest mass energy term cancels out, leaving the usually much smaller kinetic energy (and possibly potential energy terms as well). Sometimes the rest mass energy is important, *per se*, and its very large value can be a big surprise. For example, the rest mass energy of *one gram of matter* is 9×10^{20} ergs = 9×10^{13} J. This is slightly more energy than was released by the Nagasaki atomic bomb !! (Note that it is precisely this mass-energy formula that is the basis for the bomb. No other physics formula has had such an impact on humanity.)

The consequence of the mass-energy formula is that in order for a massive object to reach a velocity v it is necessary to provide enough energy to account for the γ-factor increase. For our trip to the center of the galaxy we needed $\gamma = \sqrt{5} \times 10^3$. If we take into account the mass of the space traveller, say 100 kg, and for the moment ignore (omit) the mass of the spaceship and its supplies, this means we need

$$\sqrt{5} \times 10^3 \times 10^5 \times 9 \times 10^{13} J \sim 2 \times 10^{22} J$$

This is more than the energy of 100 million Nagasaki bombs. Even if we could make a production line that gave us one bomb *per hour*, it would take more than 10,000 years to provide enough bombs. How these would be used to get the spaceship up to speed is not at all clear. Carrying any of them in the spaceship would only increase the need for many more bombs in order to get them up to speed too.

To obtain velocities so close to the speed of light it appears necessary to build an extremely massive machine. Compare the masses of the machines at Fermi Lab, at CERN, at SLAC,… to the mass of an electron ($9.10938188 \times 10^{-28}\ g$) or a proton ($1.67262158 \times 10^{-24} g$). Velocities for massive bodies that are very close to the speed of light (n > 1) are very expensive to achieve.

The reader should be able to do a similar calculation for travel to another galaxy, say only a million parsecs away from ours. This distance is much larger than the distance of the Sun from the galactic center of the Milky Way. The size of γ becomes even larger (by more than 100 fold if the traveler experiences a 10 year trip) and the needed energy is increased by the same amount (for just the 100 kg traveler). Even so, upon return the traveler's friends would have aged by over 7 million years. Would the civilization the traveler left even still exist or remember him/her?

The analysis above is meant to convey just how vast is the Universe. Without fanciful science fiction devices for which there is no sound scientific basis we are forced to realize that space travel to other galaxies is not likely to ever happen. However, we can still imagine what reality is like everywhere else in the Universe. At http://www.fefox.com/ARTICLES/Universality.pdf I have proposed a Universality IQ test. The reader is urged to take this test. If indeed you conclude the mathematics, physics and chemistry are the same everywhere in the Universe then it is also likely that biology, and life as we know it, is the same everywhere in the Universe. That means there are thinking beings elsewhere facing the same dilemmas regarding space travel that we are but also suspecting that we too exist and are doing the same.

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February 25, 2010