An Exact Value for Avogadro's number and Higher Precision Computation

With my colleague Ted Hill, we have considered making Avogadro's number exact [N_A]. Up to the time when we wrote that paper, the measured value of N_A was given by [NIST CODATA] $6.0221415 \pm 0.0000010 \times 10^{23}$. Later (on June 30, 2007) NIST published the value $6.02214179 \pm 0.00000030 \times 10^{23}$. They actually ante-dated the change to 2006.

Our key idea was that an object in the shape of a cube with n atoms on an edge would require $n \sim 10^8$. We chose to specify this eight digit number exactly. Much of the physical constant data in CODATA has a relative error with an exponent near to that of 10^{-8} . Note that for the earlier value of N_A above the relative error is $\sim 10^{-7}$ and 5×10^{-8} for the second value. Thus the cube root of N_A can be specified to the same degree of accuracy as is presently obtained by measurement and one can envisage a hypothetical cubic array of atoms that captures some sense of the largeness of this number. The cube root is a number we can almost comprehend.

The need to compute cubes of large integers is obvious. My computer does not possess a sophisticated mathematics package that can rapidly give me 24 digit numbers and do arithmetic with them. I did notice, however, that my Google browser did arithmetic automatically, but only up to thirteen digit numbers. If I typed a number into the browser window followed by "cubed" or ^3, it automatically produced the answer, at least up to thirteen digits. This was adequate for a rough estimate of N_A consistent with present day relative error. Thus entering 84446888^3 into the window yields $6.02214141 \times 10^{23}$ which is pretty close to the NIST CODATA listing. But it does not give me the precise 24 digit number.

The way around this barrier when a mathematics package software isn't available is to do arithmetic in *higher precision*. I chose to use base 10,000 instead of the usual base 10. This means that I think about 84446889 (notice that we had to switch from 84446888 to 84446889 after NIST made their change) as

$84446889 = 8444 \times 10^4 + 6889$

This enables exact calculations using the Google browser since the cube of the above number is given by the binomial identity

$$(8444 \times 10^4 + 6889)^3$$
$$= 8444^3 \times 10^{12}$$
$$+3 \times 8444^2 \times 6889 \times 10^8$$
$$+3 \times 8444 \times 6889^2 \times 10^4$$
$$+6889^3$$

and we get the following results from the browser:

$8444^3 = 602\ 066\ 792\ 384$
$3 \times 8444^2 \times 6889 = 1\ 473\ 580\ 577\ 712$
$3 \times 8444 \times 6889^2 = 1\ 202\ 214\ 187\ 572$
$6889^3 = 326\ 940\ 373\ 369$

Now just add the right number of zeroes to these exact results and do a long hand addition of the resulting four numbers. The result is

$602\ 214\ 162\ 464\ 240\ 016\ 093\ 369$

This is higher precision. I especially like the foxtail.

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